(Pages : 4)

Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, December 2017
First Degree Programme under CBCSS
COMPLEMENTARY COURSE FOR CHEMISTRY AND POLYMER CHEMISTRY
MM1331.2 : Mathematics – III
(Vector Analysis and Theory of Equations)
(2014 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

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SECTION – 1

(Answer **all** questions. **Each** question carries **1** mark.)

1. State the fundamental theorem of Algebra.
2. Find the number of real roots of the equation $x^3 - x^2 + x - 1 = 0$.
3. If $f(x) = (x-2)^4 \phi(x)$ and $\phi(x)$ is not divisible by $(x-2)$, find the multiplicity of 2.
4. If α, β, γ are the roots of the equation $x^3 + 2x^2 - x + 1 = 0$ find the equation whose roots are $-\alpha, -\beta, -\gamma$.
5. If α, β are the roots of the equation $x^2 + 3x + 4 = 0$, find the equation whose roots are $1/\alpha$ and $1/\beta$.
6. Find the domain of $\overrightarrow{r(t)}$ if $\overrightarrow{r(t)} = \left(\sqrt{3t+1}, t^2\right)$. Also find $\overrightarrow{r(t_0)}$ if $t_0 = 1$.
7. Find $\lim_{t \to 3} \left(t^2 \vec{i} + 2t \vec{j}\right)$.
8. Find $\overrightarrow{r'(t)}$ if $\overrightarrow{r(t)} = 4\vec{i} - \cos t \vec{j}$.

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9. Find divergence of $\vec{F}$ if $\vec{F}(x, y, z) = x^2y\vec{i} + 2y^3z\vec{j} + 3z\vec{k}$.

10. Define a Solenoidal vector field. Give an example.

SECTION – 2

(Answer **any eight**. **Each** question carries **2** marks.)

11. Show that the equation $x^3 + 2x^2 + 7 = 0$ has at least two complex roots.

12. If 1, 2, 3 are the roots of the cubic equation $f(x) = 0$, find $f(x)$ and hence find the equation whose roots are 1, 1/2 and 1/3.

13. If 1 and 3 are the roots of $g(x) = 0$, find $g(x)$ and hence find the equation whose roots are 1/2 and 3/2.

14. Solve $4x^3 - 24x^2 + 23x + 18 = 0$ whose roots are in Arithmetic Progression.

15. Sketch the graph and a radius vector of $\overrightarrow{r(t)} = \cos t\,\vec{i} - \sin t\,\vec{j}$; $0 \le t \le 2\pi$.

16. Find $\overrightarrow{y(t)}$ if $\overrightarrow{y''(t)} = \vec{i} + e^t\vec{j}$: $\overrightarrow{y(0)} = 2\vec{i}$ and $\overrightarrow{y'(0)} = \vec{j}$.

17. Find k(t) for the circular helix $x = a\cos t$, $y = a\sin t$, $z = ct$; $a > 0$.

18. Find the velocity and speed of a particle moving along the curve

$$\overrightarrow{r(t)} = t\vec{i} + \frac{t^2}{2}\vec{j} + \frac{t^3}{3}\vec{k} \text{ at } t = 1.$$

19. Find the divergence and curl of the vector field $\vec{F}(x, y, z) = x^2y\vec{i} + 2y^3z\vec{j} + 3z\vec{k}$.

20. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, show that (a) Curl $\vec{r} = 0$ (b) div $\vec{r} = 3$.

21. Evaluate $\int_c x^2ydx + xdy$ along the triangular path joining (0, 0), (1, 0) and (1, 2) in the counter clockwise direction.

22. Let $\vec{F}(x, y) = e^y\vec{i} + xe^y\vec{j}$, prove that $\vec{F}$ is a conservative field in the entire plane.

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SECTION – 3

(Answer **any six**. **Each** question carries **4** marks.)

23. If α, β, γ are the roots of the equation $8x^3 - 4x^2 + 6x - 1 = 0$, find the equation whose roots are $\alpha + 1/2, \beta + 1/2$ and $\gamma + 1/2$.

24. Find a real root of the equation $f(x) = x^3 - x - 1 = 0$ using bisection method, correct to four decimal places.

25. Solve the equation $x^3 - 5x + 3 = 0$, by Newton-Raphson method, correct to three decimal places.

26. Using the vector equation of the ellipse $\frac{x^2}{4} + \frac{x^2}{9} = 1$, find the curvature at the end points of the major and minor axes.

27. Suppose that a particle moves through 3-space so that its position vector at time t is $\overrightarrow{r(t)} = t\vec{i} + t^2\vec{j} + t^3\vec{k}$. Find
    i) The scalar tangential and normal components of acceleration at time t.
    ii) The scalar tangential and normal components of acceleration at time t = 1.
    iii) The vector tangential and normal components of acceleration at time t = 1.

28. Prove that curl (grad ϕ) = 0, where ϕ is a scalar point function.

29. Find the work done by the force field $\vec{F}(x, y) = x^3y\vec{i} + (x - y)\vec{j}$ on a particle that moves along the parabola $y = x^2$ from (– 2, 4) to (1, 1).

30. Let $\vec{F}(x, y) = 2xy^3\vec{i} + (1 + 3x^2y^2)\vec{j}$.
    a) Show that $\vec{F}$ is a conservative vector field in the entire x-y plane.
    b) Find the scalar potential ϕ.

31. State Green's theorem. Using the theorem, evaluate $\int_c x^2\, ydx + xdy$ along the triangular path joining (0, 0), (1, 0) and (1, 2) in the anticlokwise direction.

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SECTION – 4

(Answer **any two**. **Each** question carries **15** marks.)

32. Solve $x^4 - 2x^3 + 3x^2 + 4x - 1 = 0$, given that the product of two roots is unity.

33. Describe the Newton-Raphson method for finding the solution of the equation $f(x) = 0$. Solve the equation $x^4 + x^2 - 80 = 0$, by Newton-Raphson method, correct to three decimal places.

34. A shell fired from a cannon has a muzzle speed of 800ft/s. The barrel makes angle 45° with the horizontal, the barrel opening is assumed to be at the ground level. Find

    a) The parametric equations for the shell's trajectory

    b) Maximum height reached

    c) The horizontal distance travelled

    d) Speed of the shell at the point of impact with the ground.

35. State Stoke's theorem. Verify the theorem for the vector field $\vec{F}(x, y, z) = 2z\vec{i} + 3x\vec{j} + 5y\vec{k}$ taking σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation, and C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the x-y plane.

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